



Are corresponding points fixed?

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Abstract

Several investigators have claimed that the retinal coordinates of corresponding points shift with vergence eye movements. Two kinds of shifts have been reported. First, global shifts that increase with retinal eccentricity; such shifts would cause a flattening of the horopter at all viewing distances and would facilitate fusion of flat surfaces. Second, local shifts that are centered on the fovea; such shifts would cause a dimple in the horopter near fixation and would facilitate fusion of points fixated at extreme viewing distances. Nearly all of the empirical evidence supporting shifts of corresponding points comes from horopter measurements and from comparisons of subjective and objective fixation disparity. In both cases, the experimenter must infer the retinal coordinates of corresponding points from external measurements. We describe four factors that could affect this inference: (1) changes in the projection from object to image points that accompany eye rotation and accommodation, (2) fixation errors during the experimental measurements, (3) non-uniform retinal stretching, and (4) changes in the perceived direction of a monocular point when presented adjacent to a binocular point. We conducted two experiments that eliminated or compensated for these potential errors. In the first experiment, observers aligned dichoptic test lines using an apparatus and procedure that eliminated all but the third error. In the second experiment, observers judged the alignment of dichoptic afterimages, and this technique eliminates all the errors. The results from both experiments show that the retinal coordinates of corresponding points do not change with vergence eye movements. We conclude that corresponding points are in fixed retinal positions for observers with normal retinal correspondence. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

A primary function of the visual system is to estimate the three-dimensional positions of objects in space. This estimation problem can be divided into two parts: estimating direction and estimating distance. Here, we consider the problem of obtaining one estimate of direction for an object seen binocularly.

For a monocular observer, direction can be quantified by the azimuth and elevation (respectively, horizontal and vertical angles) of the retinal point stimulated. There is a one-to-one mapping between spherical coordinates (α , ε) of a retinal surface and perceived azimuth and elevation ($\hat{\alpha}$, $\hat{\varepsilon}$) (Hering, 1879). For a binocular observer, the mapping from retinal-image coordinates to perceived direction is more complicated. For every point in the left eye (α_L , ε_L), there is a

point in the right eye (α_R , ε_R) that is seen in the same direction (apart from monocular portions of the visual field). These pairs of points are *corresponding points*. From the retinal coordinates of corresponding points and an appropriate model of the eye's optics, we can calculate object locations that stimulate corresponding points. It is useful to know where corresponding object locations are because they are where fusion is easiest, stereoacuity is highest, and depth discrimination is best (Ogle, 1953; Blakemore, 1970; Krefling, 1974; Westheimer & McKee, 1978). It would be straightforward to predict these locations if corresponding points were fixed in retinal coordinates. The intersections of rays traced from corresponding retinal-image points out to the external scene would be the predicted corresponding object locations. Unfortunately, there are numerous reports that corresponding points are not fixed retinally. Clark (1936), Amigo (1965), Flom and Eskridge (1968), Shipley and Rawlings (1970b), Remole (1985), Robertson and Schor (1986), Kertesz and Lee (1987),

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and Fogt and Jones (1998a,b) all concluded that the retinal positions of corresponding points shift with horizontal vergence. In this paper, we re-examine the stability of corresponding retinal points.

1.1. Corresponding points and the horopter: mapping between spatial and retinal-image coordinates

To determine if corresponding retinal points are fixed, one would ideally stimulate candidate retinal points directly without going through the eyes' optics.

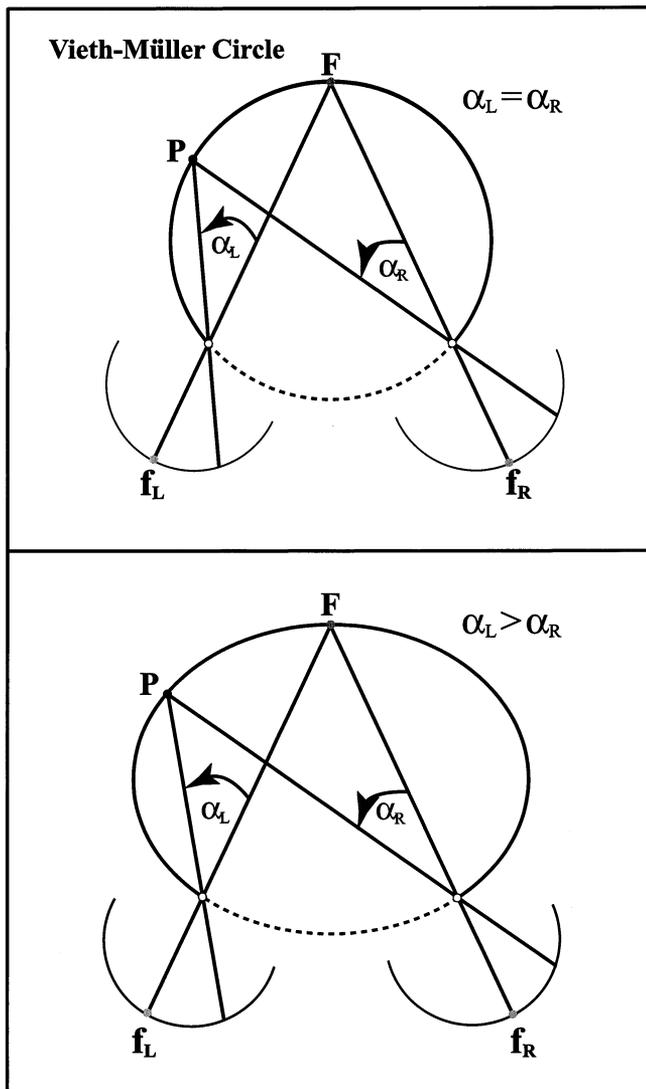


Fig. 1. Geometric and empirical horopters. The upper and lower panels are overhead views of a binocular observer fixating point, F . The foveas are represented by f_L and f_R . A second point, P , positioned to the left of the fixation point, creates the horizontal angles α_L and α_R . The disparity of P relative to F is $\alpha_L - \alpha_R$. The disparity in the upper panel is zero ($\alpha_L = \alpha_R$), so P lies on the Vieth–Müller Circle (a circle running through the fixation point and the optical centers of the two eyes). The disparity in the lower panel is positive ($\alpha_L > \alpha_R$), so P lies on an ellipse (that runs through the fixation point and the eyes' optical centers).

Because this is not possible, the positions of a pair of corresponding points have to be determined by finding the points in space that give rise to the same perceived directions in both eyes. The locations of such points define the *empirical horopter*. This horopter contains the fixation point and object points located by experimental measurement. If the angle subtended by each object point and the fixation point is the same in both eyes, the horopter is the *Vieth–Müller Circle*. The Vieth–Müller Circle is shown in the upper panel of Fig. 1; it is the circle containing the fixation point and the two eyes' optical centers. The empirical horopter and Vieth–Müller Circle lie in the visual plane that we assume projects to the horizontal meridians of the two eyes. The empirical horopter typically deviates from the Vieth–Müller Circle (Ames, Ogle, & Gliddon, 1932b; Shipley & Rawlings, 1970a,b) in the fashion depicted in Fig. 1: with increasing azimuth, object points that stimulate corresponding points lie increasingly farther behind the Circle. This deviation suggests that corresponding points along the horizontal meridians do not have the same retinal-image coordinates.

We can quantify the difference between the empirical horopter and Vieth–Müller Circle by plotting disparity as a function of azimuth (Shipley & Rawlings, 1970a,b). Alternatively, one can use conic sections to fit the empirical data and then quantify the deviation between the empirical and geometric horopters with:

$$H = \cot(\alpha_R) - L \cot(\alpha_L) \quad (1)$$

where α_L and α_R refer to azimuths with vertices at the left and right eyes, respectively (Ogle, 1932).¹ L is a skew factor and is related to the magnification of one eye's image relative to the other eye's. Measurements of corresponding points in normal observers show that disparity increases as a function of absolute azimuth and, therefore, that H is greater than 0. Equivalently, the empirical horopter is less concave than the Vieth–Müller Circle (Fig. 1).

Once we measure the empirical horopter, the retinal coordinates of corresponding points can be determined by tracing rays from horopter points onto the retinas. Obviously, one needs an appropriate model of the eyes' optics to do the ray tracing.

Many investigators (Ames et al., 1932b; Shipley & Rawlings, 1970a,b; Fogt & Jones, 1998b) have reported changes in the empirical horopter with changes in viewing distance. These changes suggest that the positions of corresponding retinal points shift (e.g. Amigo, 1965). The empirical evidence and theoretical discussions can

¹ This equation differs from Ogle's because we use the convention that positive angles are counter-clockwise.

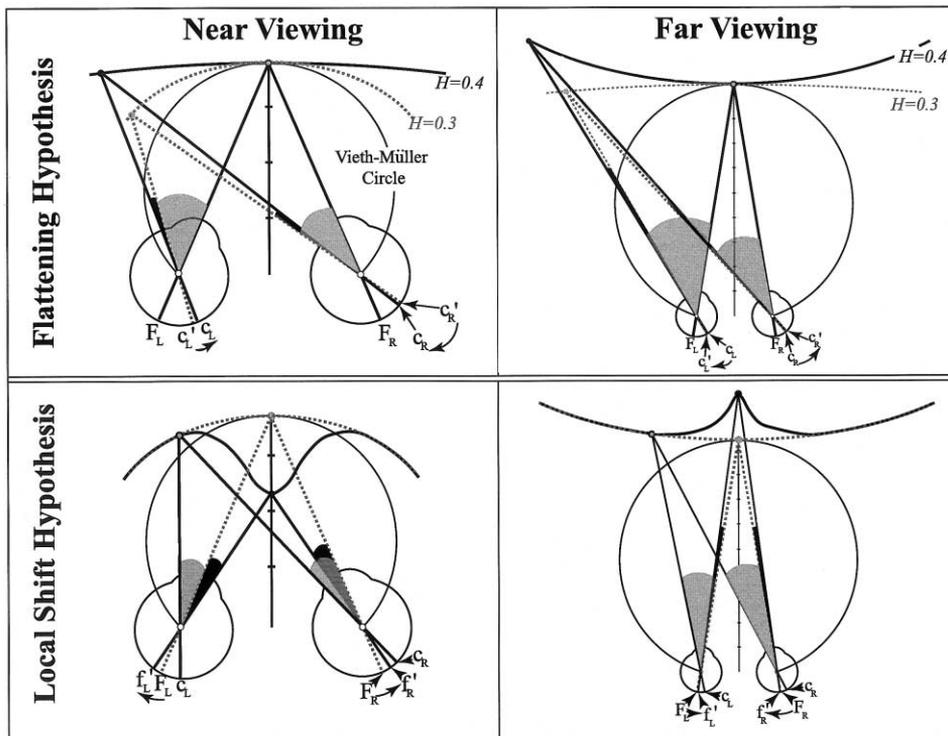


Fig. 2. Flattening and local shift hypotheses. The upper panels depict the flattening hypothesis. The foveas, F_L and F_R , are fixating a point in space. With near and far viewing, corresponding points c_L and c_R are stimulated by object points on the horopter labeled $H = 0.4$ (black line). With near viewing (left upper), that horopter is reasonably flat. With far viewing (right upper), the horopter associated with $H = 0.4$ is convex. To obtain a horopter with the same shape as the flat one at near view, H would have to decrease to a value of ~ 0.3 . Similarly, corresponding points c_L and c_R would have to shift to retinal positions c'_L and c'_R . The lower panels depict the local shift hypothesis. Again, the foveas, F_L and F_R , are fixating a point in space. The horopter for $H = 0.3$ is indicated by the dashed gray line. According to the local shift hypothesis, a shift in correspondence occurs in one direction with near viewing and in the opposite direction with far viewing. With near viewing (left lower panel), the corresponding point for F_L , the fovea of the left eye, shifts temporally in the right eye to f'_R and a similar shift occurs for the corresponding point for F_R , the fovea of the right eye. As a consequence, the horopter bows inward toward the intended fixation point as indicated by the solid-line curve. With far viewing (right lower panel), the opposite shifts occur, so the horopter bows outward toward the intended fixation point.

be categorized into two specific claims: the *flattening hypothesis* and the *local shift hypothesis*.

The flattening hypothesis states that the disparity between corresponding points increases with decreased viewing distance. If true, the change in the disparity of corresponding points would yield a flatter horopter across a range of viewing distances than fixed corresponding points. There might be an advantage to such flattening because it would make flat surfaces easier to fuse.

Predictions for the flattening hypothesis are illustrated in the upper panels of Fig. 2. The solid lines represent the horopter when $H = 0.4$ and the dashed lines the horopter when $H = 0.3$. The left and right panels show those horopters for near and far viewing, respectively. The hypothesis predicts that H decreases as viewing distance increases. This implies that corresponding retinal points c_L and c_R , measured at a near viewing distance, shift to c'_L and c'_R at a far viewing distance. These shifts, if they were appropriate for the viewing distance, would make the horopter flatter at all viewing distances than it would be if corresponding points did not shift.

The predictions are plotted in terms of disparity as a function of azimuth in the upper panel of Fig. 3. The horizontal line at 0 disparity is the Vieth–Müller Circle. If corresponding points were fixed, the disparity pattern would be the same for all viewing distances. The flattening hypothesis predicts that disparities increase as a function of azimuth more rapidly as the observer fixates closer; the three curves show such a change with fixation.

Ames et al. (1932b) measured the horopter at different fixation distances. Fig. 4 shows the results. The upper panel plots disparity as a function of azimuth, and the lower panel plots H as a function of fixation distance. As predicted by the flattening hypothesis, H decreases, and the disparity between corresponding points decreases with increased viewing distance. Shipley and Rawlings (1970b) reported similar results.

The local shift hypothesis states that corresponding points shift in and near the foveas. The shifts yield partial compensation for fixation disparity and thereby aid fusion near fixation. Predictions for this hypothesis are illustrated in the lower panels of Fig. 2. The dashed

visual lines that intersect on the Vieth–Müller Circle represent the visual axes (lines from the anatomical foveas out through the eyes' optical centers). The solid lines represent visual lines from the attempted fixation target through the eyes' optical centers. The dashed curves show the shape of the horopter for a fixed set of corresponding points. In the lower left panel (near viewing), the eyes are converged behind the attempted target, and in the lower right (far viewing), they are converged in front of the attempted target. The disparity between the solid and dashed lines is the fixation disparity. The local shift hypothesis predicts that corresponding points F_L and F_R (the anatomical foveas) shift to f'_L and f'_R when the observer attempts to look at a near or distant target. With near viewing, the point in the right eye that corresponds with the fovea in the left eye shifts temporally on the retina (and the point in the left eye corresponding with the right eye's fovea also shifts temporally). With far viewing, the reverse occurs. Points near the fovea also undergo shifts, but they are smaller. By $\sim 1.5^\circ$ retinal eccentricity, no shifts occur (Fogt & Jones, 1998b). The predicted horopters are represented by the solid curves.

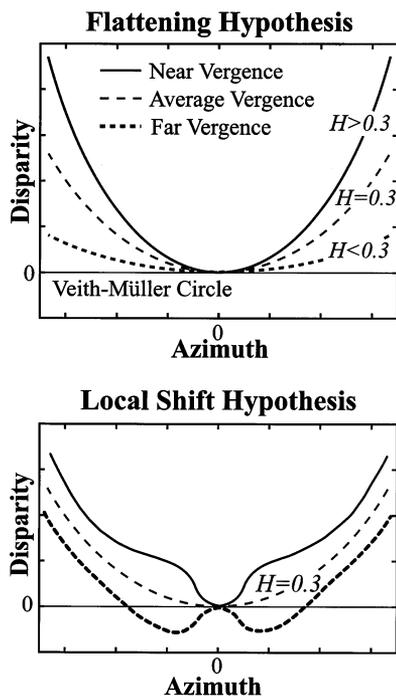


Fig. 3. Disparity of points on the empirical horopter as a function of azimuth according to the flattening and local shift hypotheses. The upper panel shows the predicted change in disparity for the flattening hypothesis. As vergence increases (distance decreases), the H value associated with the horopter increases as shown (near = solid line; average = long dashed line; far = short dashed line). The lower panel shows the predicted change for the local shift hypothesis. With near viewing, the horopter bows inward (Fig. 2), so the disparity near the shifted foveas approaches zero (with respect to the intended fixation point). With far viewing, the horopter bows outward (Fig. 2), so the disparity near the shifted foveas again approaches zero.

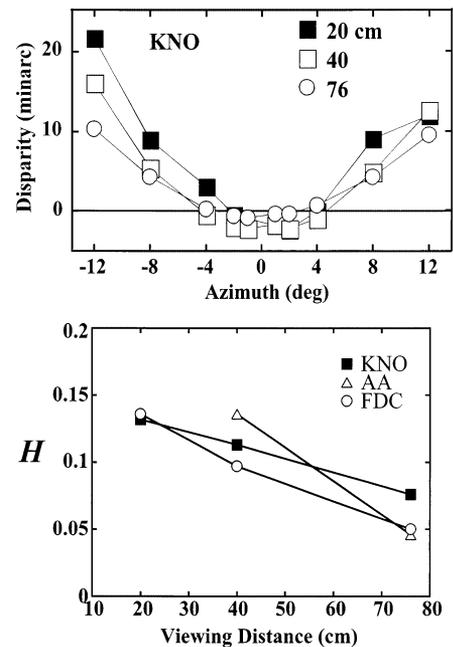


Fig. 4. Horopter data of Ames et al. (1932b). Upper panel: Disparity of the horopter settings is plotted as a function of azimuth. Different symbols represent data collected at different fixation distances. A disparity of zero (indicated by the horizontal line) corresponds with settings on the Vieth–Müller Circle (Fig. 1). Disparities greater than zero correspond with settings behind the Vieth–Müller Circle. Lower panel: H values as a function of distance. Different symbols represent data collected from different observers.

The predictions of the local shift hypothesis can also be represented as changes in disparity as a function of azimuth. The lower panel of Fig. 3 shows the predicted disparity changes at near, medium, and far fixation distances.

The main claim of the local shift hypothesis is remarkable, so it is worth emphasizing its consequences. According to Hering's Laws of perceived visual direction, when the eyes are symmetrically converged at any distance, stimulation of the foveas (F_L and F_R) yields a single percept that is straight ahead. The local shift hypothesis violates Hering's Laws. Specifically, it claims that stimulation of the foveas when the eyes are symmetrically converged at a very near distance yields the percept of two different directions, one for each eye. An object would actually have to stimulate non-foveal points (f'_L and f'_R) to be perceived in the same direction by both eyes.

The evidence for local correspondence shifts comes from comparisons of the objective and subjective measurements of eye position (Robertson & Schor, 1986; Kertesz & Lee, 1987; Fogt & Jones, 1998a,b). For example, Fogt and Jones (1998a) reported that objective measurements of vergence differed from subjective measurements by as much as 40 arcmin when the observer attempted to fixate a very near target and 50

arcmin when the observer attempted to fixate a very far target.

There are two ways to explain the global and local shifts reported in studies of corresponding points. (1) *The neural explanation*: with changes in fixation distance, the neural mapping between retinal surface coordinates and perceived direction changes. This is the explanation adopted by Amigo (1965), Flom and Eskridge (1968), Jones (1974), Nelson (1977), Remole (1985), Robertson and Schor (1986), Kertesz and Lee (1987), and Fogt and Jones (1998a,b). (2) *The optical explanation*: with changes in fixation distance, the optical mapping between object space and retinal-surface coordinates changes and not the mapping between retinal coordinates and perceived direction. This is the explanation given by Ames et al. (1932b), Ogle (1950), Bourdy (1972), Miles (1975), and Reading (1980).

To determine whether the apparent change in corresponding points is neural or optical in origin, one requires an appropriate model of the optical mapping between object points and retinal-image points. Previous work has assumed a model eye with an optical center at a fixed position relative to the center of eye rotation (Ames, Ogle, & Gliddon, 1932a; Ames et al., 1932b; von Tschermak, 1942; Fogt & Jones, 1998a,b; Amigo, 1965). Using this assumption, corresponding retinal points appear to change with viewing distance. For example, consider the data in Fig. 4. The angles used to determine disparity— $\alpha_L - \alpha_R$ —were measured with vertices at fixed nodal point positions (Ames et al., 1932a,b). Furthermore, the disparity calculations assumed that the eyes were verged accurately during the experimental measurements.

We next examine how the assumptions of a fixed optical center and accurate fixation affect the interpretation of horopter data.

1.2. Changes in positions of the nodal points

If we know the positions of the eyes' nodal points, we can map positions of points in space into retinal coordinates. The human eye has a primary and secondary nodal point. A ray passing through the primary nodal point exits from the secondary point at the same angle relative to the optical axis. Tracing these rays yields an accurate mapping from object points into retinal coordinates.

In most horopter experiments, including the Ames et al. (1932b) experiment, observers set rods in space until the parts seen by the left and right eyes appeared in the same direction. The rod settings are, therefore, defined in head-centered coordinates. To convert from these coordinates into retinal coordinates, we need to know the head-centered coordinates of the primary and secondary nodal points and the head-centered coordinates of retinal surface points.

Unfortunately, nodal-point positions in head-centered coordinates change with eye rotation and accommodation (and retinal positions change with eye position).

With eye rotation, the nodal points translate relative to the head because the nodal points are in front of the eyes' centers of rotation. Fig. 5A illustrates this effect. For simplicity, the primary and secondary nodal points are represented by one point. The upper and lower panels show nodal-point positions for far and near viewing, respectively. Notice that nodal points translate nasally with convergence. By changing the head-centered positions of the nodal points, eye rotations change the ray tracing from object points to retinal points. This effect must be eliminated or taken into account in order to determine the retinal positions of corresponding points from horopter data.

With accommodation, the positions of the nodal points shift toward and away from the cornea, and this has a small effect on the ray tracing from object points to retinal coordinates. This is illustrated in Fig. 5B. With the positive accommodation associated with near viewing (right half of panel), the primary and secondary nodal points move toward the cornea along the optical axis (the distance between them also increases slightly).² The shift toward the cornea causes a very small change in the optical projection from object to retinal points. For example, the largest change in disparity induced by a 10-diopter change in accommodation would be 1.1 arcsec at an azimuth of 6 deg; smaller changes would occur at smaller azimuths.³ Because the effect is so small, we can ignore it for the purposes of this paper.

1.3. Fixation error

Binocular fixation is typically not completely accurate, particularly at near and far fixation distances (Ames & Gliddon, 1928; Clark, 1936; Ogle, 1950; Hebbard, 1962). Observers tend to fixate nearer than distant fixation targets and farther than near ones. If such fixation errors are not eliminated or taken into account, the disparity of horopter settings (Fig. 4) will be misestimated. Fig. 6 shows how fixation errors can confound the interpretation of horopter data. The observer is converged behind the fixation target (under-

² The right panel of Fig. 5B shows that the size of the image is increased slightly with positive accommodation. A point in space, α° , from the visual axis stimulates the retinal surface coordinate, a_{far} , when accommodation is minimal. With positive accommodation, the image of the point at α° from the visual axis stimulates the retinal surface coordinate a_{near} .

³ Image position shifts by about 0.96 arcmin. The shift is in the same direction in both eyes, so the change in disparity due to accommodation is very small.

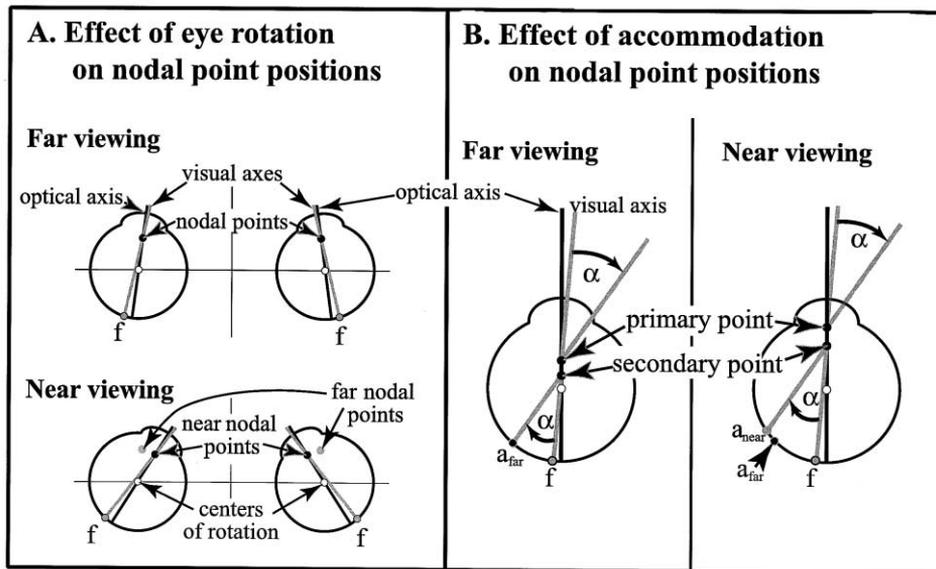


Fig. 5. Effect of eye rotation and accommodation on the optical projection from object points to retinal-image points. (A) Effect of eye rotation. The upper and lower parts are overhead views of the binocular viewing situation. The centers of rotation are represented by white dots. The nodal points (a single point is used to represent the primary and secondary points) for the depicted viewing situation are represented by black dots. The optical and visual axes are represented by black and gray lines, respectively. The foveas are represented by the gray dots labeled f . The upper part of the panel shows the positions of these points at a long viewing distance. The lower part shows the positions at a near viewing distance; the gray dots labeled 'far nodal points' represent the positions of the nodal points in the upper part of the figure. The nodal points translate nasally in head-centered coordinates when the eyes converge. (B) Effect of accommodation. The dots and lines represent the same things as in panel A with the following exceptions. The single nodal point is now replaced by primary and secondary nodal points that are represented by a pair of black dots. α represents a fixed horizontal angle between the visual axis and a ray projected from the eye. With far viewing (left part of panel), α projects to position a_{far} on the retina. With near viewing (right part of panel), α now projects to position a_{near} , which is shifted temporally relative to a_{far} . Thus, the same angle in space projects to a slightly different retinal position as the accommodative state of the eye changes.

converged). If the experimenter assumes the observer is fixating the fixation point, the measurements of the angles to horopter points will be incorrect. In the depicted case, the resulting error in the estimated disparity between corresponding points would be $\epsilon_L + \epsilon_R$. Two types of fixation error may have affected the interpretation of previous horopter data: static and transient errors. Static errors occur when the observer systematically fixates farther or nearer than the fixation target. Transient errors occur when the observer shifts eye position during an experimental measurement. Transient errors could have easily occurred in many of the previous experiments. For example, the observers in the Ames et al. (1932b) study were instructed to maintain fixation on a central point while making fine adjustments of a vertical rod presented in the retinal periphery. It seems likely that observers would have occasionally made saccades in the direction of the peripheral rod while making those settings.

1.4. The current project

We measured the horopter using techniques that circumvented the above-mentioned problems. From these data, we can determine if corresponding retinal points shift as a function of fixation distance.

2. Experiment 1

2.1. Subjects

Three observers participated. Two were aware of the experimental hypotheses, and one was not. All three had normal stereopsis and did not manifest eye misalignment in normal viewing situations.

2.2. Apparatus

2.2.1. Eliminating the effect of nodal point translation during vergence

The stimuli were displayed on a haploscope with two mirrors (one for each eye) and two CRTs (one for each eye) (for details, see Backus, Banks, van Ee, & Crowell, 1999). Each mirror and CRT was attached to an armature that rotated about a vertical axis passing through the eye's center of rotation. With this arrangement, the eye and haploscope arm rotate on a common axis, so when we change the vergence distance, the mapping between the stimulus array and the retina is unaltered (for fixed accommodation). Thus, the retinal images are unaffected by a change in the vergence of the fixation target. The upper and lower panels of Fig. 7 show the arm positions for a long and a near fixation distance, respectively.

2.2.2. Positioning the eyes

For the haploscope arrangement to achieve the desired result, the rotation axes of the haploscope arms and eyes must be co-linear. To assure that they were, we used a sighting technique. The sighting apparatus had two pairs of sight lines (Fig. 8). Each sight line consisted of two sighting targets: an aperture and a small target. One pair of sight lines—one line for each eye—was rotated clockwise so the observer had to look right to sight down them. The other pair was rotated counter-clockwise so the observer looked left to sight down them. The distance between the left- and right-eye sight lines was set equal to the observer's inter-ocular distance. With this arrangement, the sight lines intersect in two spatial positions that are an inter-ocular distance apart. The trick is to get the eyes' centers of rotation to those intersection points.

The observer's bite bar was mounted on a stand that could translate in three orthogonal directions and rotate about three orthogonal axes. The observer was translated and rotated until he reported alignment of all

Disparity Error Caused by Fixation Disparity

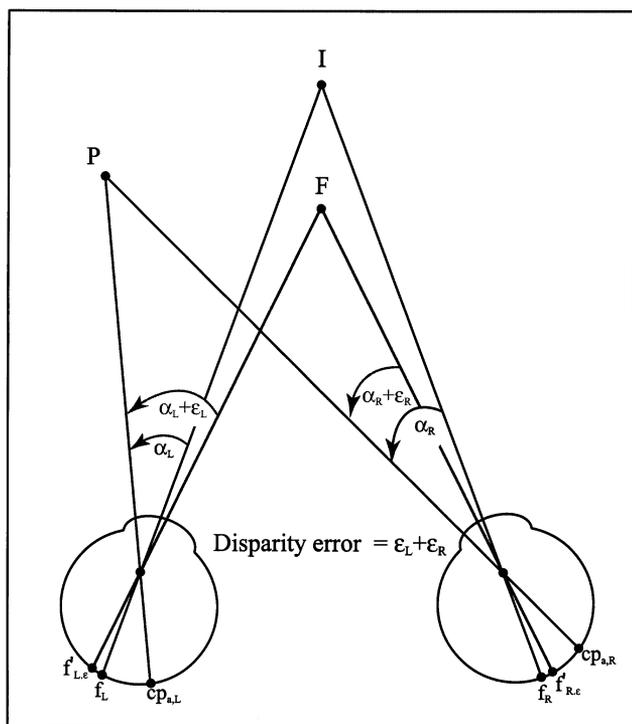


Fig. 6. Errors in assumed vergence affect the interpretation of horopter data. The observer attempts to fixate F , but is actually directing the visual axes toward point I . Point P is on the horopter (with fixation on I). The angles between the visual axes (the lines through I) and the visual lines to P are α_L and α_R for the left and right eyes, respectively. If the experimenter assumes that the eyes are directed toward F , the estimated angles will be $\alpha_L + \epsilon_L$ and $\alpha_R + \epsilon_R$ where ϵ_L and ϵ_R are the errors in assumed directions of the visual axes of the left and right eyes, respectively. The resulting error in the disparity estimate is $\epsilon_L + \epsilon_R$.

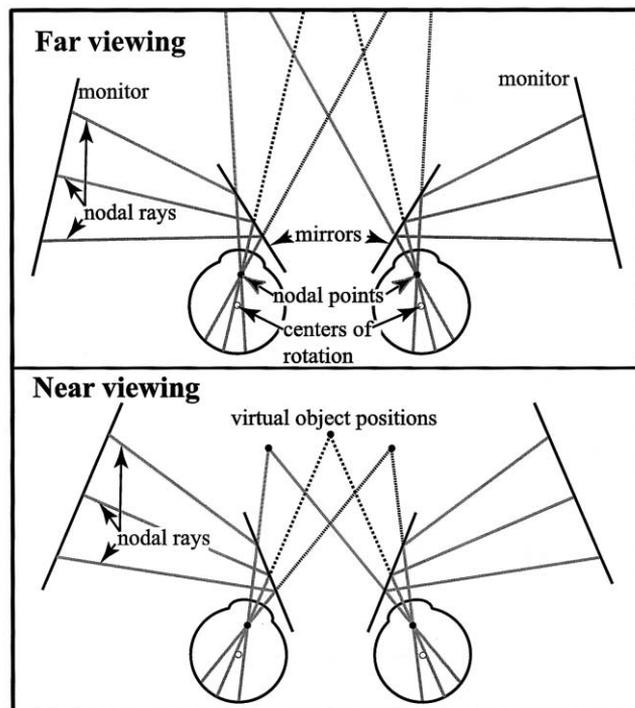


Fig. 7. Haploscope and projection of the stimuli to the two eyes at far and near fixation distances. The eyes' rotation centers are represented by the white dots and the nodal points by black dots. The haploscope arms each contain a mirror and a CRT that rotate as a rigid unit about an axis through the eye's center of rotation. The upper and lower panels show three nodal rays in each eye for far and near fixation distances, respectively. The far ends of the rays correspond to locations on the CRTs. Notice that the rays project to the same retinal points for all fixation distances (assuming no accommodation and accurate fixation).

four sight lines. That head position places the eyes' rotation centers at the intersections of the sight lines. Once the head was in the correct position, we locked the bite bar mount. The locked mount was then used to position the observer precisely in the haploscope during experimental sessions.

2.2.3. Spatial calibration

The stimuli consisted of lines and dots. The line and dot positions in our displays were specified to within 20–30 arcsec. To achieve such high spatial precision, we used anti-aliasing and a spatial calibration procedure. Each texture element in our stimuli was a dot composed of four pixels (2×2). For anti-aliasing, the intensities of the four pixels were adjusted to place the dot's apparent center at the desired location (taking into account adjacent pixel non-linearities, Klein, Hu, & Carney, 1996). To implement spatial calibration, an observer, correctly positioned in the haploscope, perceptually aligned dots on the screen with the intersections of a planar grid positioned just in front of the screen. From the resulting settings, we constructed a

look-up table that converted the desired x - y position on the grid plane into screen coordinates (Backus et al., 1999).

2.2.4. Pinhole pupils

The distance from the eye to the CRT was fixed at 39 cm, so the accommodative stimulus was fixed at 2.6 diopters. Of course, vergence can also stimulate accommodation. In pilot work, the observers were hyperopically focused with respect to the CRTs in the 172 cm vergence condition and myopically focused in the 19 cm vergence condition. The resulting blur made the Nonius task difficult to perform. To circumvent this problem, we placed pinholes on the two corneas. The pinholes allowed the observers to see the test lines clearly so they could perform the task with high precision. Observers wore opaque contact lenses with a central aperture 1.7–1.8 mm in diameter. The lenses were Alden HP Soft Contact Lenses with a power of 0 diopters.

2.3. Stimuli

It was important to hold fixation stable, so we used a specially designed fixation stimulus (Fig. 9). The large

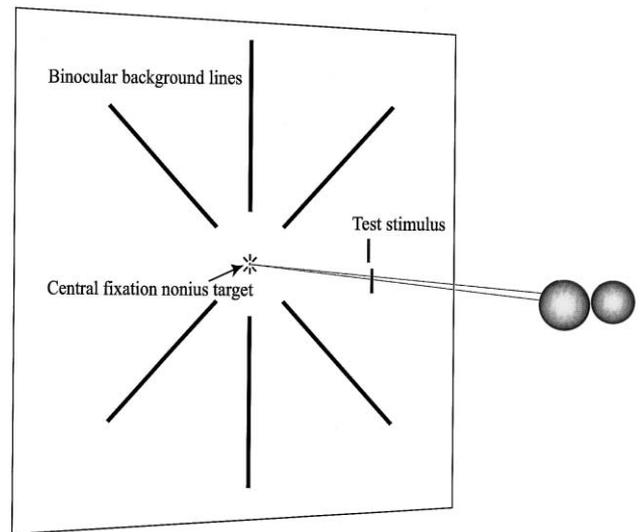


Fig. 9. Schematic of the fixation and test stimuli used in Experiment 1. The central fixation target consisted of a binocular dot (the point observers were instructed to fixate) surrounded by dichoptic Nonius lines 0.5 deg in length. The observer used the perceived alignment of the dichoptic Nonius lines to assess the accuracy of fixation. The large propeller pattern surrounding the central target consisted of six binocular lines 12.4 deg in length and was used to help maintain accurate fixation on the central target. The test stimuli consisted of dichoptic Nonius lines, one presented just above the visual plane and the other just below it. The length of these lines and the vertical separation between them varied with retinal eccentricity. Observers indicated whether the upper line appeared to be to the left or right of the lower line. The upper line was presented to the left eye half the time and to the right eye the other half.

propeller pattern and central fixation dot were binocular. The small propeller pattern surrounding the central dot was dichoptic. The top was presented to the left eye and the bottom to the right. The fixation pattern produced stable fixation near the desired plane. The fixation target was always presented straight ahead at vergence distances of 19.1 or 171.9 cm.

We used vertical Nonius test lines to measure the positions of corresponding points and fixation disparity. They were presented at one of 25 azimuths (-6° to 6°).⁴ The upper line was presented randomly, trial by trial, to the left or right eye. Observers could not tell which eye was stimulated by the upper or lower line. The gaps⁵ between the Nonius lines and the lengths of the lines varied with azimuth in order to make them roughly equally visible.

⁴ Some pilot data were collected for points beyond 6° , but the measurements were unreliable so we restricted the main experiment to smaller azimuths.

⁵ The gap sizes used were slightly less than the vertical separation that yields the lowest threshold for detecting Nonius offsets near fixation (McKee & Levi, 1987), but are also large enough to prevent fusion. For example, at azimuths of 0° and 6° , the vertical separations were 10' and 21.6'.

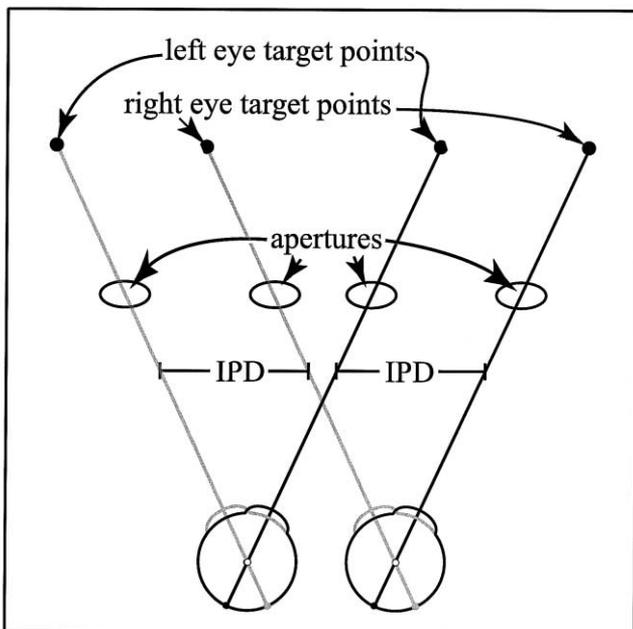


Fig. 8. Schematic of the sighting device used to position the observer. There are four sight lines. Each sight line contains a near aperture and a far point viewed through the aperture. The observer looks leftward to sight down two of the sight lines (represented by gray lines) and rightward to sight down the other two (black lines). The left and right pairs are separated by the observer's inter-pupillary distance (IPD). Two sight lines (the left gray and left black lines) intersect at one point and the other two (right gray and right black) intersect at a second point. We position the observer such that the eyes' centers of rotation are coincident with the intersections.

A Macintosh 840/AV computer generated the stimuli, controlled the staircases, and tabulated responses. Each CRT could display 1280×1024 pixels at a refresh rate of 75 Hz. The angular subtense of a pixel was 2.5 arcmin at screen center.

2.3.1. Eliminating effects of binocular targets on monocular direction

In most horopter experiments, the observer is given a binocular fixation target. Unfortunately, the presence of a binocular target within 30–60 arcmin of a monocular target influences the perceived direction of the monocular target (van Ee, Backus, & Banks, 1999). The magnitude of the influence depends on the absolute disparity of the binocular target, so the effect could vary with viewing distance (because fixation disparity varies with distance). This effect on perceived direction, which has been called binocular capture (Erkelens & van Ee, 1997) and monocular ordering (Erkelens & van de Grind, 1994), could affect measurements near the fovea, but not measurements in the parafovea or periphery. In order to eliminate intrusion due to binocular capture, we removed the fixation target immediately before the Nonius test lines were flashed. The fixation target reappeared immediately after the test lines were extinguished.⁶

2.3.2. Eliminating effects of transient and sustained fixation error

The observer was instructed to make sure that the fixation marker was aligned before initiating a stimulus presentation with a button press. Nonetheless, precise, stable alignment of the Nonius targets was difficult for some observers in some of the vergence distances. If the observer could not align the fixation targets precisely, he made sure that the apparent offset was similar from trial to trial before initiating a stimulus presentation. The test lines appeared for 100 ms in one of 25 azimuths (randomly chosen). With this brief stimulus duration, the observer could not make an eye movement toward the target; thus, systematic effects of transient fixation errors were eliminated. On each trial, the observer indicated whether the upper test line was to the left or right of the lower one. The disparity between the left- and right-eye lines was varied according to a 1-up/1-down staircase. The staircase reversed 12 times before completion; step size decreased from beginning to end. The average of the last six reversals was the estimate of the disparity that equated perceived direction for the two eyes. All azimuths were tested simultaneously by running 25 interleaved staircases. The disparity required to align the test lines at 0° was the measure of sustained horizontal fixation error.

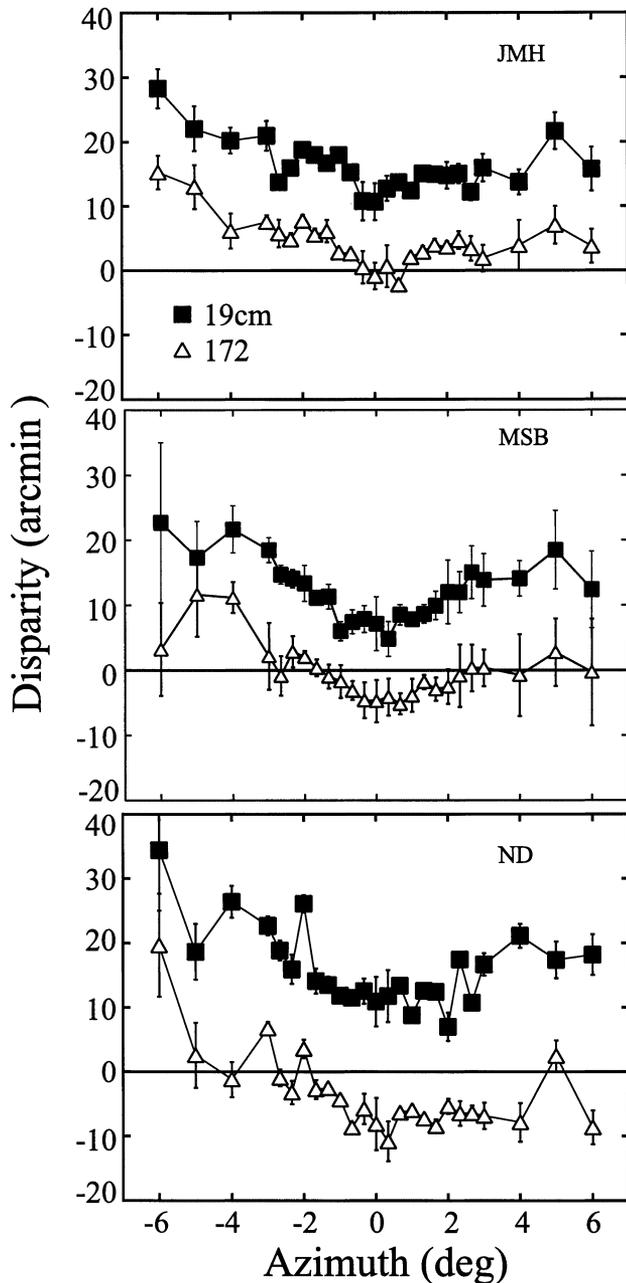


Fig. 10. Horopter settings from Experiment 1. Each panel shows data from a different observer. The disparity of the horopter settings is plotted as a function of azimuth. The horizontal lines represent the Vieth–Müller Circle. The unfilled triangles represent data collected at a viewing distance of 172 cm and the filled squares data collected at 19 cm. Error bars represent ± 1 S.D. of the last six reversals in each staircase (S.D. for each point was determined from four, two and one staircases, respectively, for MSB, JMH, and ND).

⁶ The binocular target may affect the perceived alignment of the Nonius target near fixation because the neural response associated with the binocular target may persist. One could potentially test for this possibility by adding a long blank interval between the binocular and Nonius targets. We tried this and found that observers could not obtain stable fixation. Thus, we could not make the experimental design completely immune to a potential effect of visual persistence. We believe, however, that no such effect influenced the data because the horopter curves that we obtained were smooth through the fixation point.

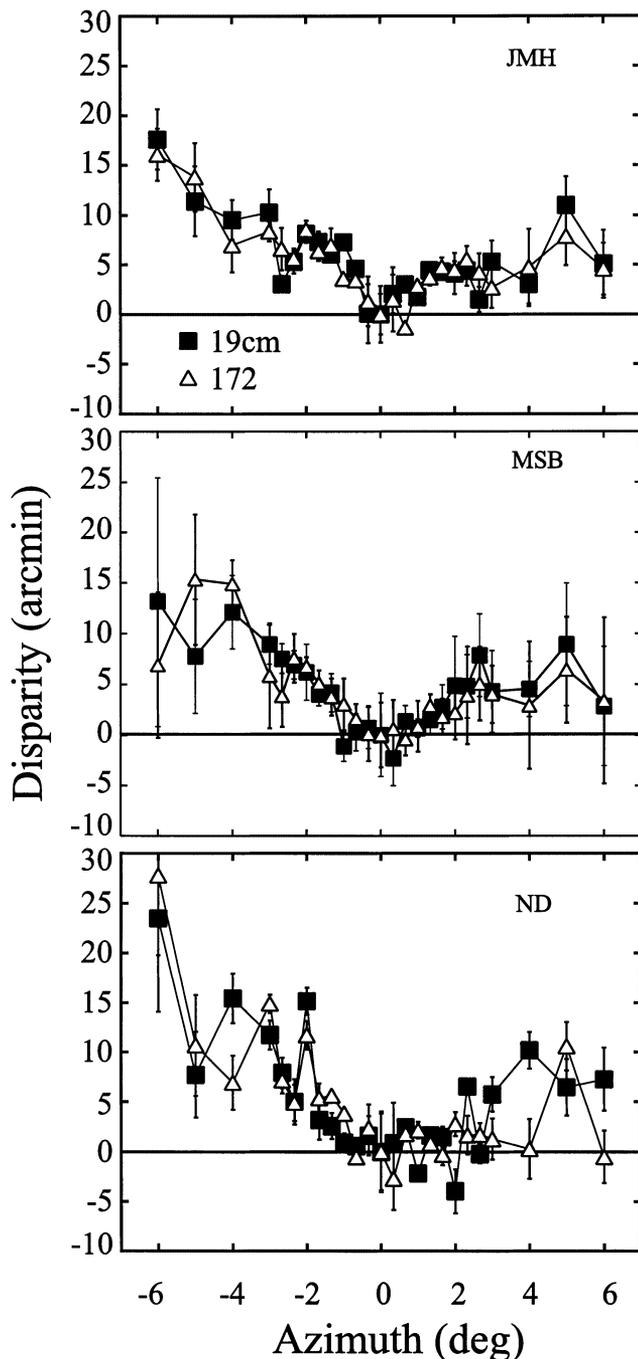


Fig. 11. Horopter settings from Experiment 1 adjusted for fixation disparity. Again, each panel shows data from a different observer. The adjusted disparity of the horopter settings is plotted as a function of azimuth. The unfilled triangles represent data collected at a viewing distance of 172 cm and the filled squares data collected at 19 cm. To adjust for fixation disparity, the functions have been shifted vertically such that the point at azimuth = 0 has a disparity of 0 deg. The required shifts were -10.7 and 0.8 arcmin for observer JMH at 19 and 172 cm, -10.9 and 8.1 for ND, and -9.1 and 3.5 for MSB. Error bars represent ± 1 S.D. as in Fig. 10.

2.4. Results and discussion

The results are summarized in Figs. 10 and 11. Each panel shows data from one observer. Fig. 10 shows the

actual disparity settings for the three observers. The settings for JMH, MSB, and ND were derived, respectively, from two, four, and one experimental sessions at each viewing distance. The vertical separation between the curves is presumably due to fixation error. Fig. 11 shows the same data after vertical displacement to superimpose at an azimuth of 0 deg.⁷ The filled and unfilled symbols represent data collected at 19 and 172 cm, respectively.

If corresponding points changed on the retina as a function of fixation distance (as suggested by the flattening hypothesis), the overall shape of the curves would change with viewing distance. There is no obvious shape change. To test the flattening hypothesis more rigorously, we found H by fitting conic sections to the data for each viewing distance. The first two columns of Table 1 list H for the two viewing distances for all three observers.⁸ The flattening hypothesis predicts that H should decrease with increasing distance. As you can see, there is no systematic (and no significant) change in H . Thus, these data offer no support for the flattening hypothesis.

Table 1
 H values for data of Experiment 1^a

	H (azimuth from -6° to 6°)		H (azimuth from -1.6° to 1.6°)	
	19 cm	172 cm	19 cm	172 cm
MSB	0.23 ± 0.12	0.24 ± 0.11	1.36 ± 0.82	1.16 ± 0.50
JMH	0.23 ± 0.09	0.22 ± 0.07	1.53 ± 0.93	1.83 ± 0.84
ND	0.35 ± 0.16	0.43 ± 0.17	1.58 ± 0.74	1.58 ± 1.57

^a Each column represents H for the conic section (Eq. (1)) that best fits the data. The three bottom rows show values obtained from the different observers. The two leftmost columns show $H \pm 95\%$ confidence for fits to all horopter points, respectively, for 19 cm data and 172 cm data. The two rightmost columns show $H \pm 95\%$ confidence for fits to horopter points between -1.6 and 1.6 deg, respectively, for 19 cm data and 172 cm data.

⁷ The choice of superimposing the 0-deg point (as opposed to other azimuths) is somewhat arbitrary. The goal is simply to bring the curves closer together so we can compare their shapes. Fixation errors at the 19 cm viewing distance for ND, JMH, and MSB were 10.9 ± 3.8 , 10.7 ± 2.9 and 9.1 ± 2.3 arcmin, respectively. Fixation errors at the 172 cm viewing distance for ND, JMH, and MSB were -8.1 ± 4.1 , -0.8 ± 2.0 and -3.5 ± 3.5 arcmin, respectively.

⁸ We used the Cartesian equations for conic sections (Ogle, 1932, p. 711) and a nonlinear least-squares fitting procedure to estimate H . The conic sections did not fit the data particularly well (which accounts for the large confidence intervals in Table 1). The inability to fit the data with these smooth curves is due to the fact that the disparity between corresponding points does not vary smoothly with azimuth. Rather, as emphasized by Shipley and Rawlings (1970a), there is idiosyncratic, but repeatable, variability in corresponding point positions.

The local shift hypothesis predicts that disparity should increase from an azimuth of 0–1.5 deg more rapidly at 19 than at 172 cm. No such change was observed. To test the local shift hypothesis more rigorously, we found H for the most relevant range of azimuths (given Fogt and Jones' description of the shifts): –1.6 to 1.6 deg. The last two columns of Table 1 list H for the two viewing distances for all three observers. There is no systematic (and no significant) change in H for all three observers. Thus, these data also offer no support for the local shift hypothesis.

We could not necessarily fix accommodation in our experiments because vergence-accommodation could have occurred. Such changes in accommodation would cause a small shift in the position of the nodal point. We are confident, however, that vergence-accommodation did not affect the data, primarily because the effect of residual accommodation would be vanishingly small.⁹

In summary, once we eliminate the confounding effects of previous experiments, there is no evidence for changes in corresponding retinal points as suggested by the flattening and local shift hypotheses. However, there is one particular shift that would not be detected by our technique: if *all* corresponding points from –6 to 6 deg azimuth shifted by the same amount, it would affect all settings by the same amount. We would be unable to detect such a shift. Notice that such a shift pattern is inconsistent with the flattening and local shift hypotheses as presented in the literature. Nonetheless, the possibility remains that a uniform shift occurred. We conducted another experiment to test this possibility.

⁹ We can reject the idea that vergence-accommodation affected our interpretation for three reasons. (1) The effect would be too small to observe. The dioptric distances associated with the nearest and farthest stimuli were 5.3 and 0.6 D, respectively. The dioptric distance to the CRTs was 2.6 D. The gain of vergence-accommodation when the accommodation stimulus is fixed at 2.6 D is less than 0.5 (Hung & Semmlow, 1980). With pinhole pupils, the gain is probably higher, but no one has made the appropriate measurements to our knowledge. None the less, the largest accommodative response that could have occurred would be between 2.4 and 4.7 D. The largest change in disparity due to this change in accommodation would be 0.26–0.50 arcsec at an azimuth of 6 deg, and this would produce a tiny and undetectable change in H . (2) Observer MSB is a presbyope, so if vergence-accommodation affected our results, it should have had the smallest effect on his data. Actually, his data show the same effect of changes in vergence, as was observed in JM and ND, which suggests that accommodation changes played no role. (3) We observed no change in the disparity of horopter settings as we changed vergence. Thus, the only way accommodation could have affected our interpretation is if it masked a change in retinal correspondence. To mask the correspondence change, the accommodation effect would have to be essentially identical, but opposite, to the effect of correspondence changes. This seems very unlikely.

3. Experiment 2

One can use afterimages to investigate the positions of corresponding retinal points as a function of viewing distance. The idea is to create an afterimage of a vertical line segment in one eye that initially appears aligned with an afterimage of a vertical line segment in the other eye; one segment should be above and the other below the fixation plane so they do not appear fused. If the local shift hypothesis is correct, or if all corresponding points shift by the same amount, dichoptic afterimages that appear aligned near the foveas should appear misaligned when vergence changes (with decreasing vergence distance, the right eye's image should appear to move rightward relative to the left eye's image). However, if corresponding retinal points are fixed, dichoptic afterimages that appear aligned at one vergence position should appear aligned at all other vergence positions. Because the afterimages are produced on the retina, the optical mapping between the external scene and the retinal image has no effect on the outcome.

Flom and Eskridge (1968) and Wick (1991) used this technique to look for correspondence shifts. Flom and Eskridge created afterimages by flashing dichoptic vertical lines, one higher than the other, onto the peripheral retinas (12, 18, and 24 deg). Observers judged the lines' perceived alignment as they changed vergence from 10 to 600 cm. The afterimages usually appeared aligned, but in some cases, they seemed to move slightly left or right relative to one another. The changes in alignment were equivalent to 3–4 arcmin at 12 deg and 6–9 arcmin at 24 deg. Wick (1991) created afterimages in a similar fashion at the fixation point. Observers made judgments of alignment as they diverged to maintain fusion of a random-dot stereogram receding in depth. Three of the six observers reported changes consistent with a shift in correspondence. Flom and Eskridge (1968) and Wick (1991) concluded that corresponding points shifted slightly with changes in fixation distance. This conclusion disagrees with the results of Experiment 1, so we decided to re-examine the perceived alignment of dichoptic afterimages.

It is not clear from their reports whether Flom and Eskridge or Wick made sure that the afterimages were initially perfectly aligned. If they were not, perceived alignment could have been affected by the well-known change in apparent image size that occurs with vergence (vergence micropsia). Consider a subject who views a slightly misaligned pair of lines with the eyes fixating far away. As he or she makes a vergence movement to fixate nearer, the afterimage will appear smaller, and so the misalignment might appear smaller as well. Thus, changes in perceived alignment of misaligned segments could be due to correspondence shifts or to vergence micropsia.

Flom and Eskridge made their measurements at retinal eccentricities of 12–24 deg. Judging alignment judgments are very difficult at those eccentricities (e.g. Levi, Klein, & Aitsebaomo, 1985), so the data were noisy. It would make more sense to make measurements at or near the fovea where alignment judgments are more accurate and where the local shift hypothesis predicts the largest correspondence shifts (Fogt & Jones, 1998b). Notice that the uniform shift (that we could not rule out in Experiment 1) predicts the same shift at the fovea as in the periphery. Again, this hypothesis is best tested at the fovea where misalignment is easiest to detect.

Wick had a binocular stimulus present while observers made alignment judgments of Nonius lines at fixation. Shimono, Ono, Saida, and Mapp (1998) showed that apparent direction of Nonius lines at fixation is influenced by the presence of binocular features, so Wick's data may have been adversely affected.

For these reasons, we repeated the afterimage study with vertical lines just above and below fixation and at 2.5° azimuth. Shifts of corresponding points on the retina will cause a change in perceived alignment of the afterimages as fixation distance changes.

3.1. Methods

The same three observers participated. A vertical strobe light was flashed to create vertical afterimages. Each line segment was 4 deg tall at the 244 cm viewing distance. Observers were dark-adapted for a few minutes before beginning the experiment.

We first measured Vernier acuity for a monocular afterimage composed of two vertical line segments. This gave us a measure of smallest discernible misalignment. The observers could reliably detect offsets of 1.5 arcmin in monocular afterimages. Therefore, the shift of 5–10 arcmin suggested by Fogt and Jones (1998b) should be easily detectable.

In the dichoptic experiment, the observer fixated a luminescent spot on the strobe in an otherwise dark room. The strobe was then flashed to one eye with the light above the fixation point and then to the other eye with the light below. Because vergence micropsia could affect judgments of the size of an existing misalignment, we made sure that the observer saw the images as perfectly aligned before conducting the rest of the experiment. Frequently, the afterimages did not appear aligned initially, so we waited several minutes until the images were no longer visible and then repeated the strobe presentation. After perfectly aligned afterimages were created, observers fixated points at three distances: less than 10 cm, 244 cm, and 700 cm. They then reported the perceived alignment of the afterimages at the three distances.

The afterimages were created in two parts of the visual field: 0 deg (just above and below the fovea) and 2.5 deg (to the right of the fixation point). Notice that the local shift hypothesis of Fogt and Jones (1998b) predicts shifts at 0 deg, but none at 2.5 deg. Ames et al. (1932b), Flom and Eskridge (1968), and Shipley and Rawlings (1970b) observed shifts at azimuths greater than 2 deg, so they would predict shifts at the 2.5 deg azimuth. A uniform shift predicts changes in perceived alignment at both 0 and 2.5 deg.

3.2. Results and discussion

All three observers reported no change in the apparent alignment of the afterimages at either azimuth. Given that they should have been able to detect alignment changes of 1.5 arcmin or larger, these data show that correspondence shifts as large as those claimed by Fogt and Jones (~15 arcmin; Fogt & Jones, 1998b) and Flom and Eskridge (3–9 arcmin; Flom & Eskridge, 1968) do not occur. Thus, these data are strong evidence that corresponding retinal points do not shift with changes in vergence. They do not shift locally, as proposed by Fogt and Jones (1998a,b) and others, and they do not shift uniformly.

4. General discussion

We find that corresponding retinal points along the eyes' horizontal meridians do not shift with vergence eye movements. Siderov, Harwerth, and Bedell (1999) concluded that corresponding points along the eyes' vertical meridians do not shift with vergence eye movements. Herzau and Ogle (1937) concluded that correspondence along the horizontal meridians does not change with horizontal version eye movements. Nonetheless, our finding directly contradicts several previous studies (Flom & Eskridge, 1968; Shipley & Rawlings, 1970b; Remole, 1985; Robertson & Schor, 1986; Kertesz & Lee, 1987, 1988; Fogt & Jones, 1998a,b). Here, we discuss the likely causes of the disagreement. There are four potential sources of error in interpreting horopter or fixation disparity data: movement of the nodal points with vergence, fixation error, retinal stretch, and effects of nearby binocular stimuli on perceived direction.

4.1. The eye's optics

Calculating corresponding retinal points from horopter data requires an appropriate model of the eye's optics. Most previous work on correspondence as a function of fixation distance has not taken into account the translation of the nodal points as the eyes rotate (Ames et al., 1932a,b; Fogt & Jones, 1998a,b).

We can calculate the expected change in H if correspondence is fixed, and nodal-point translation is not taken into account. Ames et al. (1932a,b) used a fixed nodal-point position 7.3 mm behind the cornea (see Fig. 5B) and a fixed distance between the two eyes' nodal points (Fig. 5A). By our calculation, they should have observed a decrease in H of 0.03 for an increase in viewing distance from 20 to 76 cm. This predicted change is in the direction of, but is smaller than, the observed change (Fig. 4). Thus, this error probably caused some, but not all, of the H change observed by Ames and colleagues.

We eliminated this problem in Experiment 1 by using an optical set-up that maintained the same optical projection to the retinas as the eyes rotated. We eliminated the problem in Experiment 2 by using an afterimage technique.

4.2. Fixation error

Calculating corresponding retinal points from horopter data also requires knowledge of where the eyes were fixated when the measurements were made. Most observers tend to fixate farther than a near target and nearer than a far target (e.g. Ogle, Martens, & Dyer, 1967), so a failure to measure fixation error can confound assessments of correspondence at different distances. Flom and Eskridge (1968), Reading (1980), and Shipley and Rawlings (1970b) acknowledged the importance of correcting fixation errors in interpreting horopter data. They showed that the distance-dependent changes in the horopter settings of Ames et al. (1932b) were reduced after correction for fixation error.

Some previous work on correspondence near the fovea measured fixation disparity subjectively and objectively and found that disparity differed in the two cases (Hebbard, 1962; Kertesz, Hampton, & Sabrin, 1983; Robertson & Schor, 1986; Remole, Cole, Matyas, McLeod, & White, 1986; Kertesz & Lee, 1987, 1988; Reading, 1989; Fogt & Jones, 1998a). However, the two measurements did not necessarily measure fixation disparity when the eyes were in the same state. The demands on the observer differ during objective and subjective measurements. During objective measurements, the observer fixates a binocular target, and an instrument records eye position over some time interval. In the Fogt and Jones (1998a) experiment, the objective measure was the average vergence over a 2 s interval. During subjective measurements, the observer is actively involved in a perceptual task (e.g. judging the alignment of two lines), so he/she knows precisely when the judgment is being made and, therefore, when to make the greatest effort to verge the eyes. In the Fogt and Jones (1998a) experiment, the observer made a Nonius line setting and, therefore, only needed to hold the eyes at the desired vergence position long enough to

make the perceptual judgment. Because the observer is likely to report the subjective appearance when the greatest vergence effort is being made, we suspect that fixation is more accurate in subjective than in objective measurements. This concern about the comparability of objective and subjective measures of fixation disparity applies to Hebbard (1962), Kertesz et al. (1983), Kertesz and Lee (1987, 1988), Remole et al. (1986), Robertson and Schor (1986), Reading (1989), and Fogt and Jones (1998a,b).¹⁰

In Experiment 1, we took fixation error into account by estimating fixation disparity subjectively during the course of the experimental measurements; specifically, we measured the eyes' vergence at the same time as the correspondence measurements were made. Fixation error could not affect the results of Experiment 2 after the afterimages were created.

4.3. Retinal stretching

Muscle forces that occur during accommodation cause stretching of the retina (Aubert, 1857; Hensen & Voelckers, 1873; Jaensch & Schonheinz, 1924; Moses, 1970). Specifically, contraction of the ciliary muscle causes the leading edge of the retina to advance by as much as 0.5 mm. Non-uniform stretching would change the mapping between object points and the retina; Blank and Enoch (1973) showed that it can affect perceived direction. If the stretching differed in the two eyes, the accommodation that accompanies vergence could also affect horopter settings.

This potential source of error could have affected the interpretation of our data in Experiment 1, but apparently, it did not because we observed fixed correspondence across a wide range of viewing distances. The absence of a stretching effect is probably due to the fact that we did not present accommodation demands nearly as large as Blank and Enoch (1973). Retinal stretching would have no effect on the interpretation of Experiment 2 because the percepts were created by afterimages.

¹⁰ Another problem has arisen in comparisons of objective and subjective measures of fixation disparity (Remole et al., 1986; Robertson & Schor, 1986; Kertesz & Lee, 1987, 1988; Fogt & Jones, 1998a,b). An appropriate comparison would use the same point in the eye as the vertex of the measured angles. The eye trackers used for objective measurements measure the angular rotation of the eyes relative to a fixed vertex, which is generally the eye's center of rotation. The vertex in subjective measurements is the eye's nodal point. The nodal points are closer to the cornea than the center of rotation is, so objective and subjective methods measure different angles. The difference is small, but probably accounts for some of the difference between subjective and objective methods observed in the range of normal viewing distances (Remole et al., 1986; Robertson & Schor, 1986; Kertesz & Lee, 1987, 1988; Fogt & Jones, 1998a).

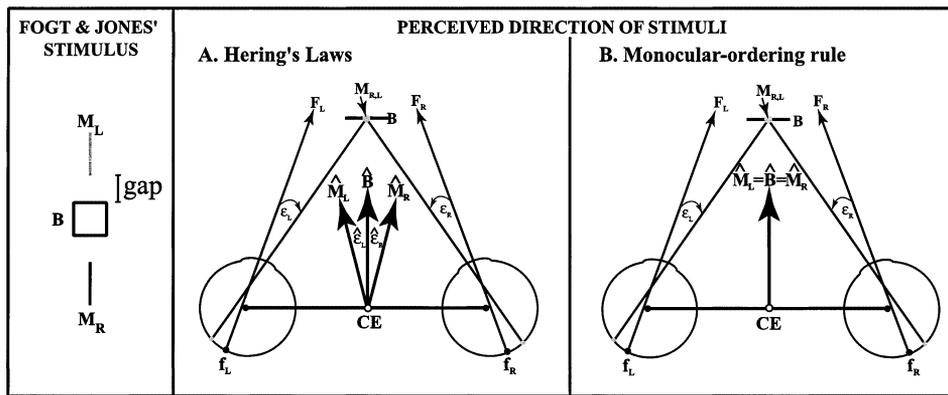


Fig. 12. Effect of nearby binocular stimuli on the perceived direction of a monocular stimulus. The left panel is a schematic of the Fogg and Jones' (1998b) stimulus. The binocular square that served as the fixation target is represented by B . The Nonius lines presented to the left and right eyes are represented by M_L and M_R , respectively. The gap between the binocular square and the Nonius lines was varied in the experiment. The two panels on the right depict perceived directions of the Nonius lines according to different rules: Hering's Laws (A) and the monocular-ordering rule (B). The left and right eyes are shown along with the cyclopean eye (CE). The observer is fixating behind the binocular box (B). The visual axes are indicated by the rays labeled F_L and F_R . The angles between the visual axes and the center of B are indicated by ϵ_L and ϵ_R . The Nonius lines M_L and M_R are positioned such that M_L is physically aligned with B , as seen by the left eye, and M_R is physically aligned with B , as seen by the right eye. According to Hering's Laws, M_L and M_R should be seen in different directions than B . Those directions are indicated by the rays \hat{M}_L and \hat{M}_R , which differ from the perceived direction, \hat{B} , by the angles ϵ_L and ϵ_R . According to the monocular-ordering rule, the perceived directions of adjacent monocular and binocular stimuli are assigned according to the angles in the eye that sees both. Thus, these rules predict that \hat{M}_L , \hat{M}_R , and \hat{B} will all be seen in the same direction.

4.4. Effects of nearby binocular stimuli on perceived direction.

In most studies of correspondence at the fovea, a binocular fixation target is presented adjacent to monocular Nonius lines (Hebbard, 1962; Bourdy, 1973; de Decker & Scheffel, 1978; Remole, 1985; Kertesz & Lee, 1987, 1988; Fogg & Jones, 1998a,b). The presence of a binocular target within 30–60 arcmin of a monocular target influences the perceived direction of the monocular target (van Ee et al., 1999). Specifically, the perceived directions of the monocular and binocular stimuli appear to be assigned by the eye that sees both. This behavior of monocular ordering (or binocular capture) violates Hering's Laws of perceived direction (Erkelens & van de Grind, 1994). With increasing separation between the binocular and monocular targets, there is a smooth transition from monocular ordering to Hering's Laws (van Ee et al., 1999). The differences in the forms of behavior are illustrated in the middle and right panels of Fig. 12. We tried to eliminate the influence of binocular capture in Experiment 1 by removing the fixation target just before the Nonius test lines were flashed.

The binocular capture effect probably affected the interpretation of many previous studies. Consider, for example, the stimulus used by Fogg and Jones (1998b) that is depicted in the left panel of Fig. 12. Observers fixated the binocular square and aligned the monocular Nonius lines above and below the square. Fogg and Jones found that settings varied as a function of gap size. According to van Ee et al. (1999), the monocular-

ordering rule applies to the calculation of perceived direction when the gap is small, and Hering's Laws apply when the gap is large. van Ee et al. reported a smooth transition from the monocular-ordering rule to Hering's Laws of perceived direction as a function of gap size. Can this model plus an assumption of fixed retinal correspondence predict Fogg and Jones' results? Or does one have to assume that corresponding points shift on the retinas to explain their results? To find out, we derived the predictions of van Ee et al. (1999) for the conditions of Fogg and Jones' (1998b) first experiment. The averaged results of four observers from van Ee et al. (their Fig. 9) were used to determine the weights given to Hering's Laws and monocular ordering for each gap size. The weights reflect how well the different rules of perceived direction predict the results from van Ee et al. These weights were then used to predict the perceived directions of Nonius lines in Fogg and Jones (1998b).

The predictions of Hering's Laws and monocular ordering are schematized in Fig. 12. The visual axes are represented by F_L and F_R , the binocular target by B , and the monocular Nonius lines by M_L and M_R . The monocular and binocular targets are superimposed in space. The binocular target is presented at a very near distance, so the observer is fixating behind the target. The deviations of the left and right eyes are represented by the angles ϵ_L and ϵ_R , respectively. According to Hering's Laws, the fused binocular square should be perceived in the head's median plane because the perceived direction, \hat{B} , of a fused binocular point is the average of the directions reported by the two eyes. The

monocular Nonius lines should be seen in different directions, \hat{M}_L and \hat{M}_R , as shown. According to the monocular-ordering rule, the perceived directions of the monocular line seen by the left eye should be the same as the direction of the binocular target because, according to these rules, the assigned directions preserve the angles seen by the left eye. The same argument applies to the line seen by the right eye. Thus, when the objects are superimposed in space, they should be seen in the same direction even when the observer is not fixating in the plane of the binocular stimulus.

Fig. 13 plots Fogt and Jones' (1998b) data and the predictions of the model. The angle between the Nonius line settings is plotted as a function of the separation between the binocular and monocular stimuli. The monocular-ordering predictions are represented by the dashed horizontal line at 0 deg. Settings based on monocular ordering could lead one to believe that there was no fixation disparity. The predictions of van Ee et al. (1999) and Hering's Law are represented by the upper and lower solid lines, respectively. We used the objective eye-position measurements from Fogt and Jones (1998b) to derive those predictions. They wiggle

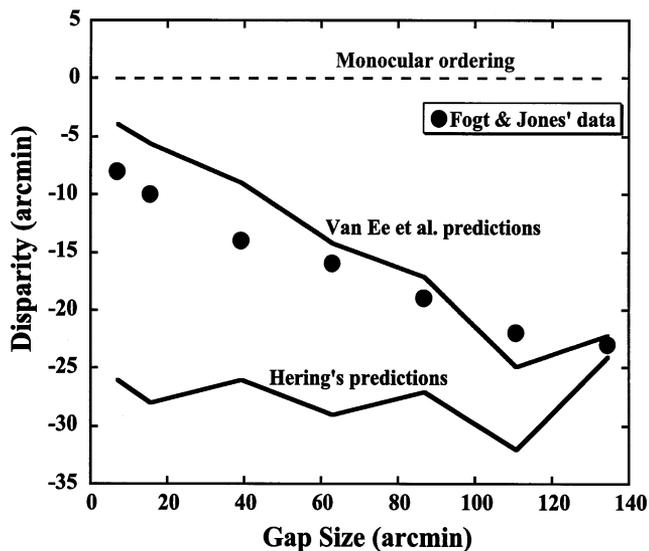


Fig. 13. Fogt and Jones' (1998b) data and the predictions of Hering's Laws, the monocular-ordering rule, and the model of van Ee et al. (1999). The observed and predicted disparities of alignment settings are plotted as a function of the gap between the binocular box and monocular Nonius lines (left side of Fig. 13). Fogt and Jones' data are represented by the filled circles. In calculating the predictions of the van Ee et al. model (gray line), we assumed that no shift in retinal correspondence occurs and that Fogt and Jones' objective measurements of eye position were correct. The monocular-ordering rule predicts that all settings should have a disparity of zero; this is indicated by the horizontal dashed line. Hering's Laws predict that the settings will vary with fixation disparity; they are represented by the black line. The van Ee et al. (1999) model predicts that settings will vary significantly with gap size because of changes in eye position and because the weight given to monocular ordering as opposed to Hering's Laws varies with gap size.

slightly because objective position varied with gap size. Fogt and Jones' subjective data are represented by the filled circles. The model predicts the data reasonably well. Thus, an apparent difference can occur between objectively and subjectively measured fixation disparity without a shift in the retinal positions of corresponding points or a change in eye position.

4.5. Strabismus

Our observers had normal binocular vision with no measurable eye misalignment. Their behavior was completely consistent with fixed retinal correspondence. There have been numerous reports that strabismic exhibit correspondence shifts that depend on eye position (e.g. Flom, 1980; Schor, 1991). The most persuasive observation is that strabimics with anomalous retinal correspondence show changes in the perceived directions of dichoptic afterimages as they move their eyes. Thus, our conclusion does not apply to patients with anomalous retinal correspondence. It would be interesting to know whether strabimics with normal retinal correspondence truly behave as if their corresponding points are fixed.

5. Conclusion

Since the beginning of the 20th century, investigators have asked whether corresponding points are fixed in retinal coordinates. Much of the data has suggested that corresponding points shift along the retinas with changes in fixation distance. This conclusion is not necessarily justified unless one can eliminate or take into account alterations in the projection of horopter points in space to corresponding locations on the two retinas. Four effects can alter this relationship: (1) changes in the projection from object to image points that accompany eye rotation and accommodation, (2) fixation errors during the experimental measurements, (3) non-uniform retinal stretching, and (4) changes in perceived direction of a monocular point when presented adjacent to a binocular point. We eliminated the influence of these four effects and found no evidence for shifts in the retinal coordinates of corresponding points. Thus, we conclude that corresponding points have fixed positions on the two retinas.

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